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A SINGLE AND LINEAR CONTROL DESIGN FOR A CLASS OF NONLINEAR SYSTEMS WITH UNKNOWN PARAMETERS AND UNCERTAIN ACTUATOR NONLINEARITIES

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ABSTRACT
In this paper, the robust stabilization for a class of nonlinear systems with unknown parameters and uncertain actuator nonlinearities is investigated. Based on differential and integral inequalities, a simple linear control is offered to realize the global exponential stabilization of such uncertain systems. Besides, the guaranteed exponential decay rate can be correctly calculated. Finally, several numerical simulation results will be provided to demonstrate the correctness and effectiveness of the main results.
KEYWORDS: *Robust stabilization, Uncertain systems, Global exponential stabilization, Exponential decay rate*

1. INTRODUCTION

As we know, there are more or less uncertain factors in real physical systems. These uncertainties may come from unknown noise, incomplete models, or uncertain system parameters. If these uncertain factors are not taken into account, the designed controller often cannot achieve the expected goal, and even cause system instability or oscillation. Furthermore, if these uncertain factors are considered, the design of the controller becomes extremely difficult because the model is too complex.

In recent years, the design of robust controllers for uncertain systems has been explored and proposed by many researchers; see, for example, [1]-[10] and the references therein. Various methodologies in the robust control have been proposed, such as Lyapunov approach, adaptive control, linear matrix inequalities, variable structure control, fuzzy adaptive control design strategy, and others.

This paper considers the problem of controller design for a class of nonlinear systems with both unknown parameters and uncertain actuators. Using differential and integral inequalities, a simple hardware-implemented linear controller is designed to promote such uncertain systems to achieve the global exponential stability. Meanwhile, the guaranteed exponential decay rate can be correctly calculated. Finally, several numerical simulation results will be provided to show the correctness and effectiveness of the main theorem. Throughout this paper, $|a|$ denotes the modulus of a real number *a* and $\|x\|$ denotes the Euclidean norm of the vector $x \in \Re^n$.

2. PROBLEM FORMULATION AND MAIN RESULTS

Consider the following uncertain nonlinear systems with unknown parameters and uncertain actuator nonlinearities described by

$$\dot{x}_1 = \Delta a(t)x_1 + \Delta d_1(t)x_2 + f_1(x_1, x_2, x_3, x_4), \tag{1a}$$

$$\dot{x}_2 = \Delta d_2(t)x_1 + \Delta d_3(t)x_2 + \Delta d_4(t)x_3 + \Delta d_5(t)x_4 + f_2(x_1, x_2, x_3, x_4) + \Delta\phi(u), \tag{1b}$$

$$\dot{x}_3 = \Delta d_6(t)x_2 + \Delta b(t)x_3 + f_3(x_1, x_2, x_3, x_4), \tag{1c}$$

$$\dot{x}_4 = \Delta d_7(t)x_2 + \Delta c(t)x_4 + f_4(x_1, x_2, x_3, x_4), \quad \forall\, t \ge 0, \tag{1d}$$

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where $x(t) := [x_1(t) \quad x_2(t) \quad x_3(t) \quad x_4(t)]^T \in \Re^{4\times 1}$ is the state vector, $u \in \Re$ is the control input, $\Delta a(t), \Delta b(t), \Delta c(t),$ and $\Delta d_i(t)$ are unknown time-varying parameters, f_i is nonlinear term with $f_i(0,0,0,0) = 0, \forall i \in \{1,2,3,4\}$, and the operator of $\Delta\phi(u) : \Re \to \Re$, is the uncertain actuator nonlinearity. In addition, in order to ensure that the uncertain systems of (1) have solution, we assume that $f_1, f_2, f_3,$ and f_4 are smooth functions.

Throughout this paper, we make the following assumptions:

(A1) There exist constants $\overline{a}, \underline{a}, \overline{b}, \underline{b}, \overline{c}, \underline{c},$ and $\overline{d_i}$ such that

$$-\overline{a} \le \Delta a(t) \le -\underline{a} < 0, \quad -\overline{b} \le \Delta b(t) \le -\underline{b} < 0,$$

$$-\overline{c} \le \Delta c(t) \le -\underline{c} < 0, \quad |\Delta d_i(t)| \le \overline{d_i}, \quad \forall i \in \{1,2,3,4,5,6,7\}.$$

(A2) There exists a positive number r_1 such that uncertain actuator nonlinearity satisfies

$$r_1 \cdot u^2 \le u \cdot \Delta\phi(u).$$

(A3) $\sum_{i=1}^{4} x_i \cdot f_i(x_1, x_2, x_3, x_4) = 0$.

The definition of global exponential stabilization for the uncertain systems (1) is as follows.

Definition 1: If there exist a control *u* and positive number α satisfying

$$\|x(t)\| \le \|x(0)\| \cdot e^{-\alpha t}, \quad \forall t \ge 0,$$

the uncertain system (1) is said to be globally exponentially stable. At the same time, the positive number α is called the exponential decay rate.

The purpose of this paper is to design a suitable control *u* to ensure global exponential stability of the system (1). Besides, we will figure out the exponential decay rate of the uncertain system.

Now we present the main result for the global exponential stabilization of uncertain systems (1) via differential and integral inequalities.

Theorem 1: The uncertain systems (1) with (A1)-(A3) is globally exponentially stable under the following linear controller

$$u = -\left(\frac{\alpha_1 + \alpha_2 + \alpha_3 + \alpha_4 + \overline{d_3}}{r_1}\right) x_2, \tag{2a}$$

where

$$\alpha_1 = \frac{(\overline{d_1} + \overline{d_2})^2}{\underline{a}}, \quad \alpha_2 = \frac{(\overline{d_4} + \overline{d_6})^2}{\underline{b}}, \quad \alpha_3 = \frac{(\overline{d_5} + \overline{d_7})^2}{\underline{c}}, \tag{2b}$$

with any $\alpha_4 > 0$. In this case, the guaranteed exponential decay rate is calculated as

$$\alpha := \min\left\{\frac{3\underline{a}}{4}, \quad \frac{3\underline{b}}{4}, \quad \frac{3\underline{c}}{4}, \quad \alpha_4\right\}. \tag{3}$$

Proof: Let

$$V(x(t)) := x_1^2(t) + x_2^2(t) + x_3^2(t) + x_4^2(t). \tag{4}$$

The time derivative of $V(x(t))$ along the trajectories of the closed-loop systems (1) with (2)-(3) and (A1)-(A3), is given by

$$\begin{aligned}
\dot{V}(x(t)) &= 2x_1\dot{x}_1 + 2x_2\dot{x}_2 + 2x_3\dot{x}_3 + 2x_4\dot{x}_4 \\
&= 2x_1(\Delta a x_1 + \Delta d_1 x_2 + f_1) \\
&\quad + 2x_2[\Delta d_2 x_1 + \Delta d_3 x_2 + \Delta d_4 x_3 + \Delta d_5 x_4 + f_2 + \Delta\phi(u)] \\
&\quad + 2x_3(\Delta d_6 x_2 + \Delta b x_3 + f_3) + 2x_4(\Delta d_7 x_2 + \Delta c x_4 + f_4) \\
&\le -2\underline{a}x_1^2 + 2\overline{d_1}|x_1||x_2| + 2\overline{d_2}|x_1||x_2| + 2\overline{d_3}x_2^2 + 2\overline{d_4}|x_2||x_3| + 2\overline{d_5}|x_2||x_4| \\
&\quad + 2\overline{d_6}|x_2||x_3| - 2\underline{b}x_3^2 + 2\overline{d_7}|x_2||x_4| - 2\underline{c}x_4^2 + 2(x_1 f_1 + x_2 f_2 + x_3 f_3 + x_4 f_4) \\
&\quad + 2x_2\Delta\phi(u) \\
&= -2\underline{a}x_1^2 + 2(\overline{d_1} + \overline{d_2})|x_1||x_2| + 2\overline{d_3}x_2^2 + 2(\overline{d_4} + \overline{d_6})|x_2||x_3| + 2(\overline{d_5} + \overline{d_7})|x_2||x_4| \\
&\quad - 2\underline{b}x_3^2 - 2\underline{c}x_4^2 + 2x_2\Delta\phi(u) \\
&= -2\underline{a}x_1^2 + 2(\overline{d_1} + \overline{d_2})|x_1||x_2| + 2\overline{d_3}x_2^2 + 2(\overline{d_4} + \overline{d_6})|x_2||x_3| + 2(\overline{d_5} + \overline{d_7})|x_2||x_4| \\
&\quad - 2\underline{b}x_3^2 - 2\underline{c}x_4^2 - \frac{2r_1 u\Delta\phi(u)}{\alpha_1 + \alpha_2 + \alpha_3 + \alpha_4 + \overline{d_3}} \\
&= -2\underline{a}x_1^2 + 2(\overline{d_1} + \overline{d_2})|x_1||x_2| + 2\overline{d_3}x_2^2 + 2(\overline{d_4} + \overline{d_6})|x_2||x_3| + 2(\overline{d_5} + \overline{d_7})|x_2||x_4| \\
&\quad - 2\underline{b}x_3^2 - 2\underline{c}x_4^2 - \frac{2r_1^2 u^2}{\alpha_1 + \alpha_2 + \alpha_3 + \alpha_4 + \overline{d_3}} \\
&= -2\underline{a}x_1^2 + 2(\overline{d_1} + \overline{d_2})|x_1||x_2| + 2\overline{d_3}x_2^2 + 2(\overline{d_4} + \overline{d_6})|x_2||x_3| + 2(\overline{d_5} + \overline{d_7})|x_2||x_4| \\
&\quad - 2\underline{b}x_3^2 - 2\underline{c}x_4^2 - 2(\alpha_1 + \alpha_2 + \alpha_3 + \alpha_4 + \overline{d_3})x_2^2 \\
&= -2\left(\frac{3\underline{a}}{4}x_1^2 + \alpha_4 x_2^2 + \frac{3\underline{b}}{4}x_3^2 + \frac{3\underline{c}}{4}x_4^2\right) - 2\left[\frac{\underline{a}}{4}x_1^2 - (\overline{d_1} + \overline{d_2})|x_1||x_2| + \alpha_1 x_2^2\right] \\
&\quad - 2\left[\frac{\underline{b}}{4}x_3^2 - (\overline{d_4} + \overline{d_6})|x_2||x_3| + \alpha_2 x_2^2\right] - 2\left[\frac{\underline{c}}{4}x_4^2 - (\overline{d_5} + \overline{d_7})|x_2||x_4| + \alpha_3 x_2^2\right] \\
&= -2\left(\frac{3\underline{a}}{4}x_1^2 + \alpha_4 x_2^2 + \frac{3\underline{b}}{4}x_3^2 + \frac{3\underline{c}}{4}x_4^2\right) - 2\left[\frac{\sqrt{\underline{a}}}{2}|x_1| - \sqrt{\alpha_1}|x_2|\right]^2 - 2\left[\frac{\sqrt{\underline{b}}}{2}|x_3| - \sqrt{\alpha_2}|x_2|\right]^2 \\
&\quad - 2\left[\frac{\sqrt{\underline{c}}}{2}|x_4| - \sqrt{\alpha_3}|x_2|\right]^2 \\
&\le -2(\alpha x_1^2 + \alpha x_2^2 + \alpha x_3^2 + \alpha x_4^2) \\
&= -2\alpha V, \quad \forall\, t \ge 0.
\end{aligned}$$

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Hence, one has

$$e^{2\alpha t}\cdot\dot{V}+e^{2\alpha t}\cdot 2\alpha V=\frac{d}{dt}\left[e^{2\alpha t}\cdot V\right]\leq 0,\quad \forall\, t\geq 0.$$

It results

$$\int_0^t \frac{d}{d\tau}\left[e^{2\alpha\tau}\cdot V(x(t))\right]d\tau=e^{2\alpha t}\cdot V(x(t))-V(x(0))\leq\int_0^t 0\,d\tau=0,\quad \forall\, t\geq 0. \tag{5}$$

From (4) and (5), it follows

$$\|x(t)\|^2=V(x(t))\leq e^{-2\alpha t}V(x(0))=e^{-2\alpha t}\|x(0)\|^2,\quad \forall\, t\geq 0.$$

As a consequence, we conclude that

$$\|x(t)\|\leq e^{-\alpha t}\|x(0)\|,\quad \forall\, t\geq 0.$$

This completes the proof. □

Remark 1: It is worth mentioning that the proposed controller of (2) is not only a linear controller, but only a single controller can achieve the goal of global exponential stability.

3. NUMERICAL SIMULATIONS

The following examples are presented to illustrate the usefulness of the proposed theoretical results.

Example 1: Consider the uncertain systems (1) with

$$f_1=-x_1x_2x_3x_4,\quad f_2=5x_1^2x_3x_4,\quad f_3=-3x_1^2x_2x_4,\quad f_4=-x_1^2x_2x_3, \tag{6a}$$

$$\underline{a}=3,\quad \underline{b}=2,\quad \underline{c}=4,\quad \overline{d_i}=1,\quad \forall\, i\in\{1,2,3,4,5,6,7\}, \tag{6b}$$

$$\Delta\phi(u)=\Delta d_8 u+\Delta d_9 u^3,\quad \Delta d_8\geq 1,\quad \Delta d_9\geq 0. \tag{6c}$$

By choosing the parameter $r_1=1$, (A2) is obviously satisfied. From (2), we have $\alpha_1=\frac{4}{3},\alpha_2=2,\alpha_3=1$. With the choice $\alpha_4=1$, from (2) and (3), it can be readily obtained that $u=\frac{-19}{3}x_2$ and $\alpha=1$. As a consequence, by Theorem 1, we conclude that the uncertain system (1) with (6) and the linear control $u=\frac{-19}{3}x_2$ is globally exponentially stable. Furthermore, the guaranteed exponential decay rate is calculated as $\alpha=1$. Typical state trajectories for the uncontrolled system and the feedback-controlled system are shown in Figure 1 and Figure 2, respectively. In addition, the control signal and electronic circuits to realize this control law are shown in Figure 3 and Figure 4, respectively.

Example 2: Consider the uncertain systems (1) with

$$f_1=f_4=0,\quad f_2=-x_1x_3,\quad f_3=x_1x_2, \tag{7a}$$

$$\underline{a}=12,\quad \underline{b}=2.1,\quad \underline{c}=0.2,\quad \overline{d_i}=1,\quad \forall\, i\in\{3,4,5,6\} \tag{7b}$$

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$$\Delta\phi(u)=\Delta d_8 u+\Delta d_9 u^3, \quad \Delta d_8 \geq 1, \quad \Delta d_9 \geq 0. \tag{7c}$$

By choosing the parameter $r_1 = 1$, (A2) is obviously satisfied. From (2), we have $\alpha_1 = 102.1, \alpha_2 = 1.9, \alpha_3 = 245$. With the choice $\alpha_4 = 1$, from (2) and (3), it can be readily obtained that $u = -351x_2$ and $\alpha = 0.15$. Consequently, by Theorem 1, we conclude that the uncertain system (1) with (7) and the linear control $u = -351x_2$ is globally exponentially stable. Furthermore, the guaranteed exponential decay rate is calculated as $\alpha = 0.15$. Typical state trajectories for the uncontrolled system and the feedback-controlled system are shown in Figure 5 and Figure 6, respectively. In addition, the control signal and electronic circuits to realize this control law are shown in Figure 7 and Figure 8, respectively.

CONCLUSIONS

In this paper, the robust stabilization for a class of nonlinear systems with unknown parameters and uncertain actuator nonlinearities has been explored. Based on differential and integral inequalities, a simple linear control has been offered to realize the global exponential stabilization of such uncertain systems. Besides, the guaranteed exponential decay rate can be correctly calculated. Finally, several numerical simulation results have been provided to show the correctness and effectiveness of the main results.

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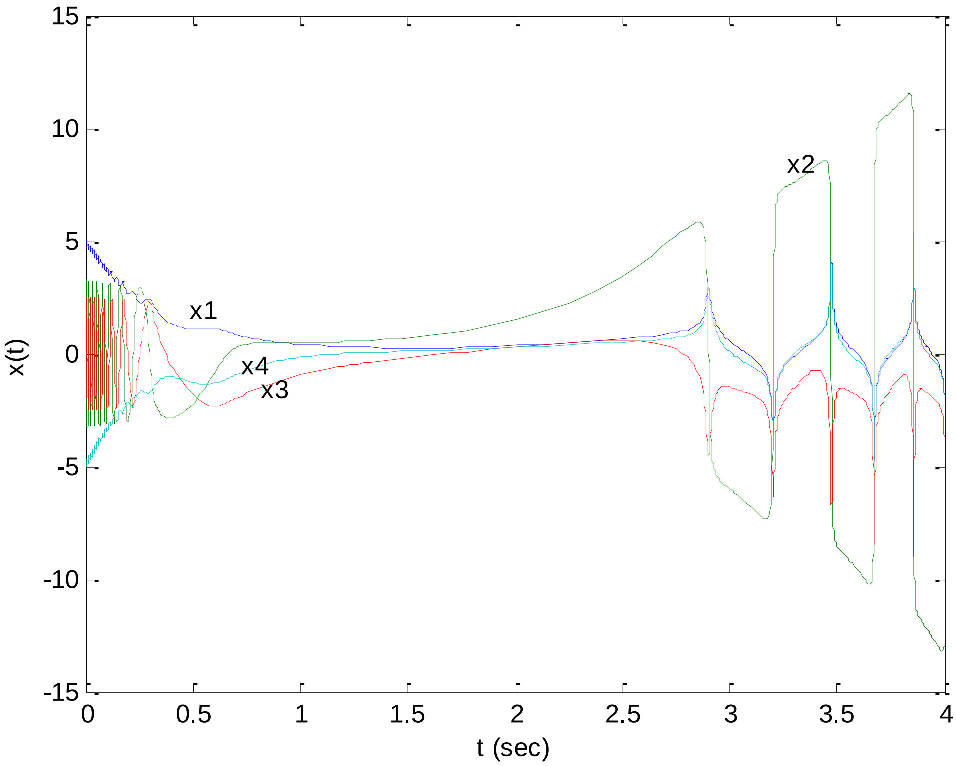

Figure 1: Typical state trajectories of the uncontrolled system of Example 1.

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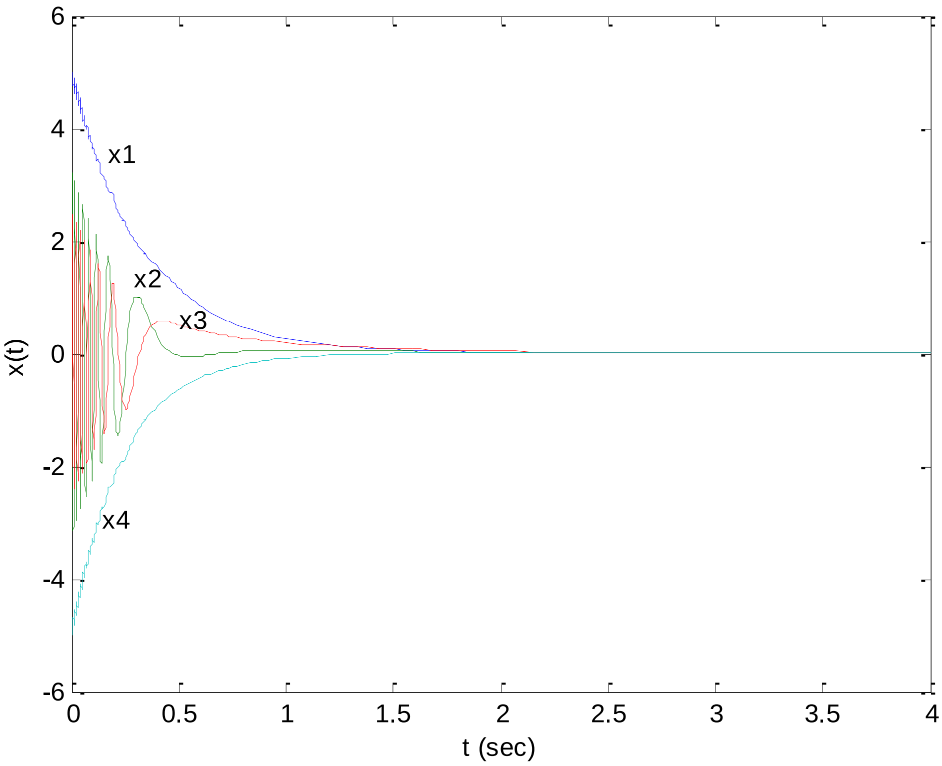

Figure 2: Typical state trajectories of the feedback-controlled system of Example 1.

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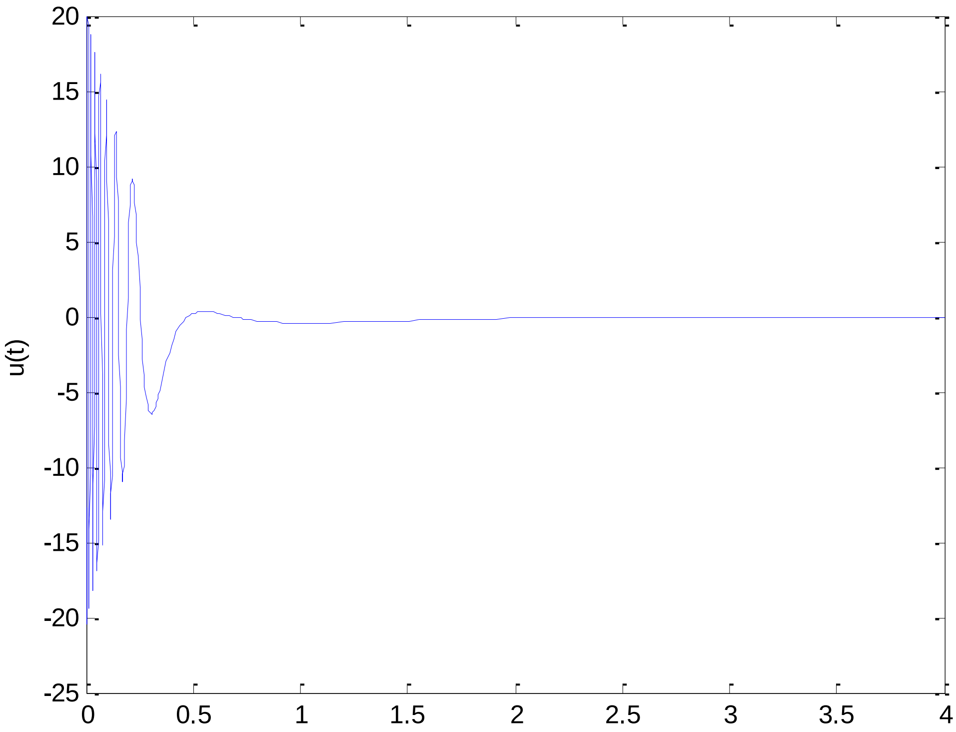

Figure 3: Control signal of Example 1.

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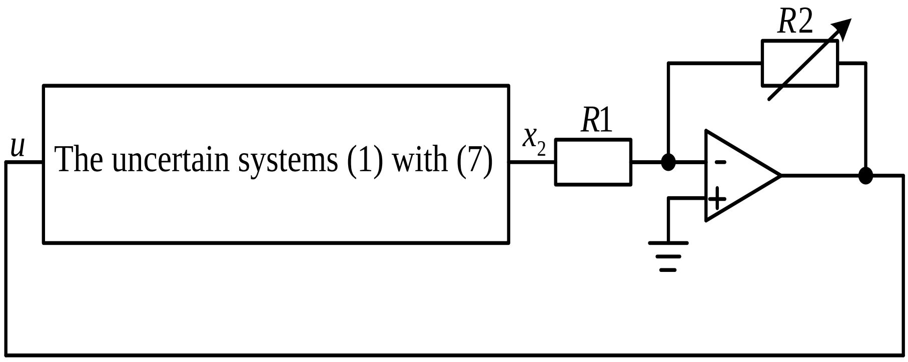

Figure 4: The diagram of implementation of Example 1, where $R1 = 3k\Omega, \quad R2 = 19k\Omega.$

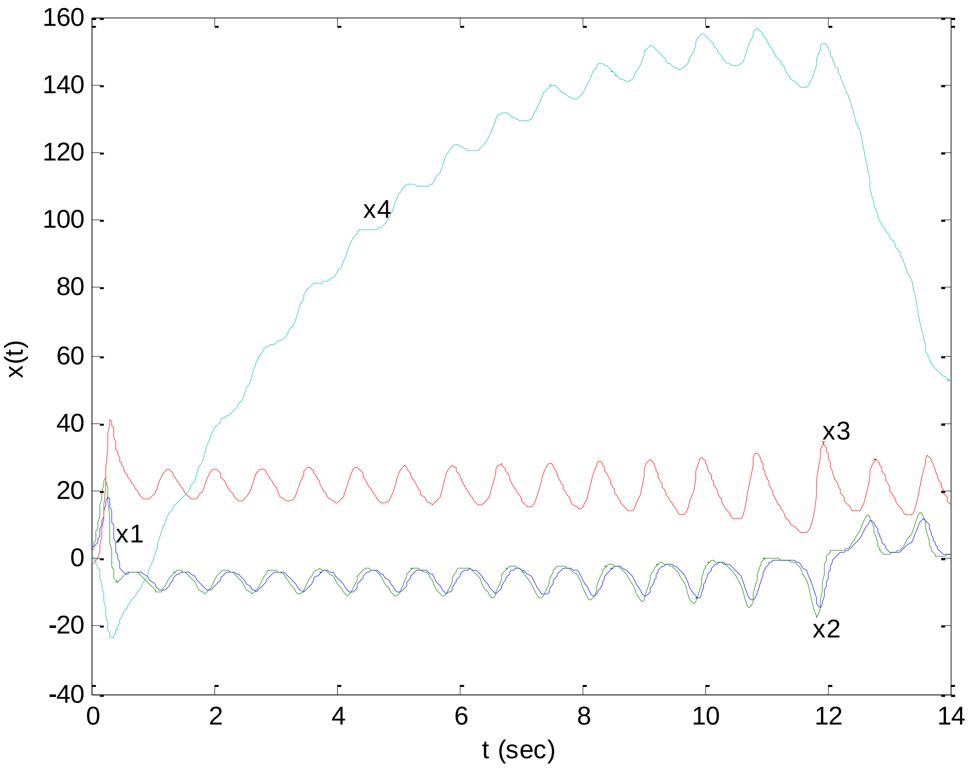

Figure 5: Typical state trajectories of the uncontrolled system of Example 2.

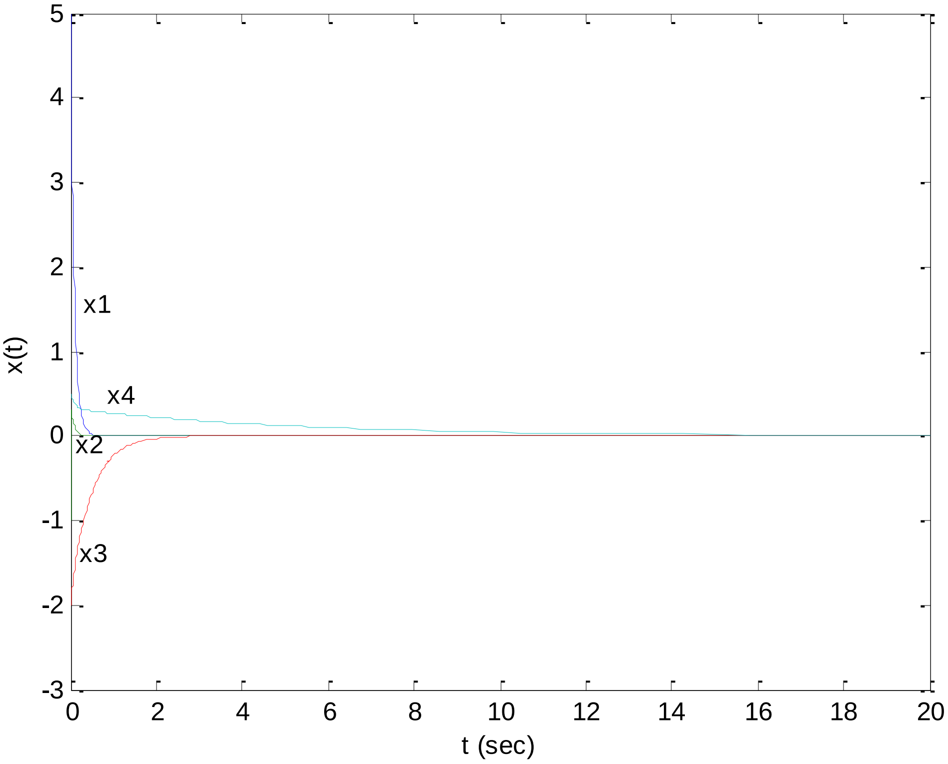

Figure 6: Typical state trajectories of the feedback-controlled system of Example 2.

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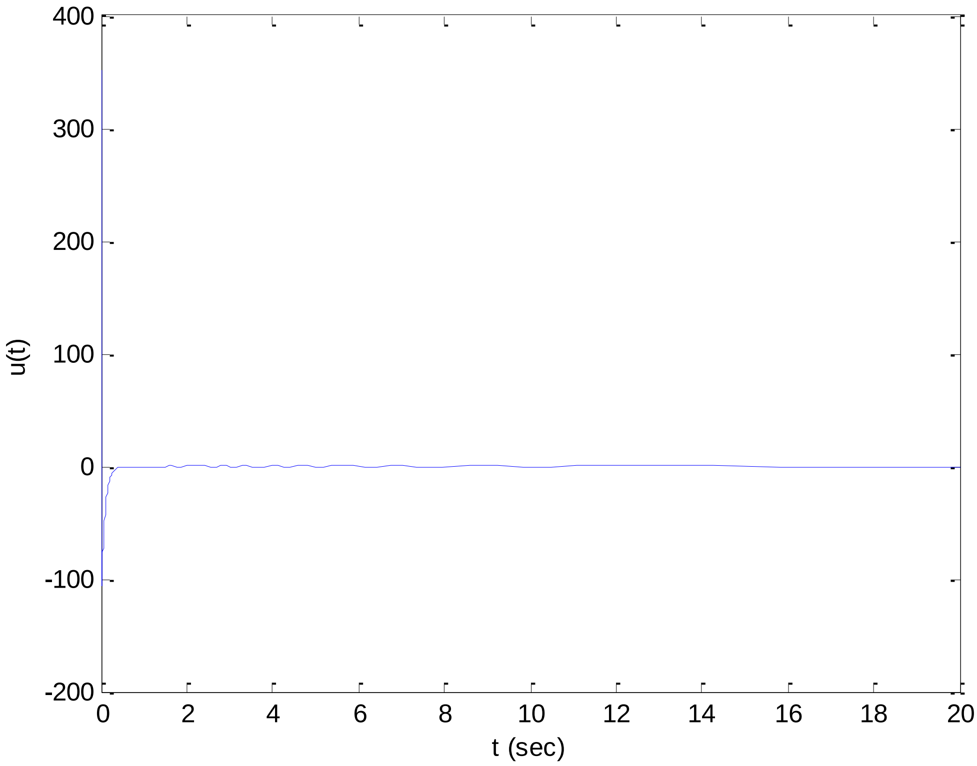

Figure 7: Control signal of Example 2.

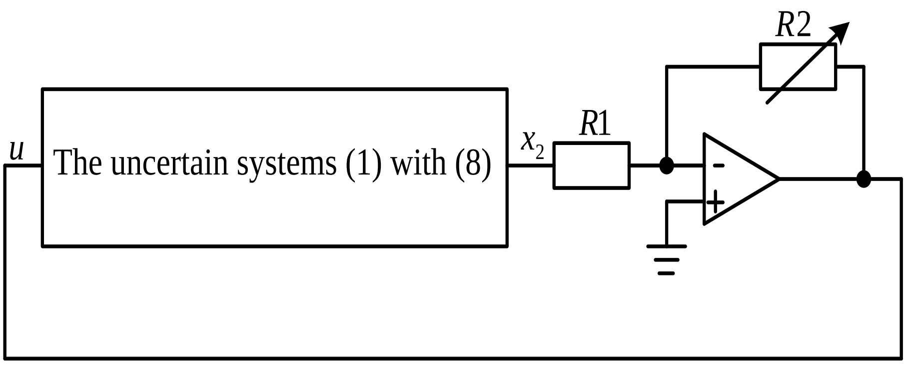

Figure 8: The diagram of implementation of Example 2, where $R1 = 1k\Omega, \quad R2 = 351k\Omega.$